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MHD oseenlet and double wake of a rising sphere under strong vertical magnetic field

Kazuyuki Ueno[1], *Hideyuki Yasuda*[2]

[1] *Department of Aerospace Engineering, Tohoku University, 980-8579 Sendai, JAPAN*
[2] *Department of Adaptive Machine Systems, Osaka University, 565-0871 Suita, JAPAN*

Flow field of electrically conducting fluid around a rising sphere is discussed. An asymptotic solution at large Hartmann number *Ha* is proposed. A function with a singularity called MHD oseenlet is derived. It gives double wake unique to MHD flows. Its flow rate across horizontal plane is a finite value in contrast with divergent flow rate for stokeslet and zero flow rate for source doublet. We obtain the flow field around a sphere in a series of MHD oseenlet and its derivatives. Fluid in conical-shape near wake moves with the rising sphere. The vertical height $\pm 0.1 Ha\, a$ of the near wake is much larger than its radius a. In far wake, the flow field is reduced to MHD oseenlet.

1. Introduction It is often required to prevent movement of droplets or particles on processing of certain types of alloys. For monotectic alloys in which liquid-liquid separation occurred, it has been generally recognized that homogeneous solidified structure was hardly obtained by conventional solidification processing, since liquid particles easily segregated due to the density difference and interfacial energy difference at solidifying front [1]. Microgravity condition is used to prevent the gravitational segregation during the solidification. For example, homogeneous solidified structure was produced under the microgravity condition to improve the superconducting properties [2].

Applying uniform magnetic field is one of the promising methods to suppress movement of particles or droplets in molten metal (Fig. 1). Even if a particle moves parallel to the magnetic field, the flow around it traverses the magnetic field. Consequently, Lorentz force acts on the flow. Therefore, the terminal velocity of a rising or sedimental particle is a function of magnetic flux density.

Theoretical prediction of the terminal velocity U_0 of a sphere under a uniform

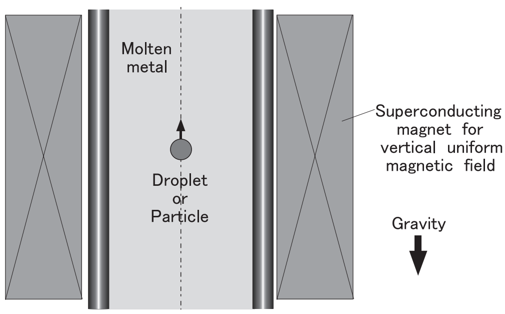

Figure 1: Rising sphere in a molten metal under a vertical uniform magnetic field.

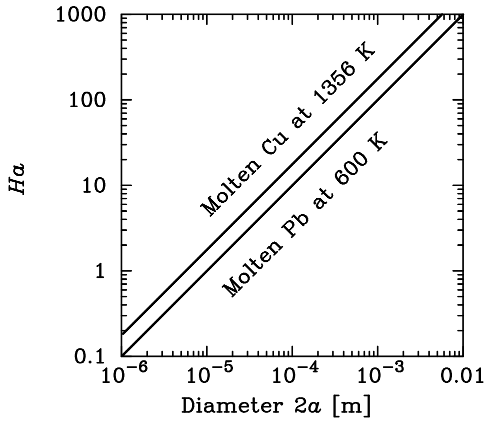

Figure 2: Hartmann number *Ha* of the flow around a sphere under 10 [T] magnetic flux density.

magnetic field is $\frac{2}{3}a^2 g(\rho_\ell - \rho_s)\eta^{-1} Ha^{-1}\left[1 + O(Ha^{-1})\right]$ when Hartmann number *Ha* is much larger than 1 [3]-[5]. Here, g, a, ρ_s, η, ρ_ℓ, denote the gravity, radius and density of the sphere, viscosity and density of the fluid, respectively. Hartmann number *Ha* is defined as

$$Ha = \sqrt{\sigma/\eta}\ B_0 a, \tag{1}$$

where B_0, σ denote magnetic flux density and electric conductivity of the fluid. This terminal velocity is a $O(Ha^{-1})$ small value compared with the terminal velocity without magnetic field. For example, 10T magnetic field dramatically decrease the terminal velocity of rising spheres in lead when the sphere diameter is greater than 0.1mm (Fig. 2).

We discuss the flow field around a rising sphere at large Hartmann number, small Reynolds number and small magnetic Reynolds number. Considering a very small sphere compared with the vessel, we neglect the effect of the vessel walls. Chester *et al.* [3],[4] studied the same problem and predict the drag of the sphere. I. D. Chang [5] gave a flow field solution including an integral of Bessel functions. Furthermore, Kyrlidis *et al.* [6] showed flow field solutions by numerical simulation. In this paper, we reconstruct the flow field solution in a series of a singularity (MHD oseenlet) and its derivatives.

2. Balance of forces We consider a flow of an electrically conducting fluid when magnetic Reynolds number ($Rm = \sigma\mu_0 U_0 a$) is much smaller than 1. Consequently, the induced magnetic field by the current in the fluid is negligibly small compared with the imposed magnetic field. The imposed magnetic field is static and uniform. Its direction is aligned with the gravity. It is denoted by $(0, 0, B_0)$ in the cylindrical coordinate system $\hat{r}\hat{\theta}\hat{z}$. The governing equations for axisymmetric flow are given as follows.

$$\frac{1}{\hat{r}}\frac{\partial \hat{r}\hat{u}_r}{\partial \hat{r}} + \frac{\partial \hat{u}_z}{\partial \hat{z}} = 0, \tag{2}$$

$$-\frac{\partial \hat{p}}{\partial \hat{r}} + \eta\left[\frac{1}{\hat{r}}\frac{\partial}{\partial \hat{r}}\left(\hat{r}\frac{\partial \hat{u}_r}{\partial \hat{r}}\right) + \frac{\partial^2 \hat{u}_r}{\partial \hat{z}^2} - \frac{\hat{u}_r}{\hat{r}^2}\right] + \hat{\jmath}_\theta B_0 = 0, \tag{3}$$

$$-\frac{\partial \hat{p}}{\partial \hat{z}} + \eta\left[\frac{1}{\hat{r}}\frac{\partial}{\partial \hat{r}}\left(\hat{r}\frac{\partial \hat{u}_z}{\partial \hat{r}}\right) + \frac{\partial^2 \hat{u}_z}{\partial \hat{z}^2}\right] - \rho_\ell\, g = 0, \tag{4}$$

$$\hat{\jmath}_r = 0, \qquad \hat{\jmath}_\theta = -\sigma \hat{u}_r B_0, \qquad \hat{\jmath}_z = 0, \tag{5}$$

where $(\hat{u}_r, 0, \hat{u}_z)$, $(0, \hat{\jmath}_\theta, 0)$ and $\hat{p}$ denote velocity, current density and pressure, respectively. Considering a flow at small Reynolds number ($Re = \rho_\ell U_0\, a/\eta \ll 1$), we neglect inertia terms in the momentum equation (3), (4). Current density has only θ component, and there is no electric field. Therefore, conductivity of the sphere has no effect to the flow. Boundary conditions at the sphere surface and infinity are given as follows.

$$\begin{array}{lll} \hat{u}_r = 0, \quad \hat{u}_z = U_0 & \text{at} & \hat{r}^2 + \hat{z}^2 = a^2, \\ \hat{u}_r \to 0, \quad \hat{u}_z \to 0 & \text{for} & \hat{r}^2 + \hat{z}^2 \to \infty. \end{array} \tag{6}$$

Let us introduce dimensionless variables:

$$\hat{r} = a\tilde{r}, \quad \hat{z} = a\tilde{z}, \quad \hat{u}_r = U_0\tilde{u}_r, \quad \hat{u}_z = U_0\tilde{u}_z, \quad \hat{p} = p_0 - \rho_\ell\, gz + \sigma a {B_0}^2 U_0 \tilde{p}. \tag{7}$$

We have the following dimensionless equations from Eqs. (2)–(6).

$$\frac{1}{\tilde{r}}\frac{\partial \tilde{r}\tilde{u}_r}{\partial \tilde{r}} + \frac{\partial \tilde{u}_z}{\partial \tilde{z}} = 0, \tag{8}$$

$$-\frac{\partial \tilde{p}}{\partial \tilde{r}} + \frac{1}{Ha^2}\left[\frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{r}}\left(\tilde{r}\frac{\partial \tilde{u}_r}{\partial \tilde{r}}\right) + \frac{\partial^2 \tilde{u}_r}{\partial \tilde{z}^2} - \frac{\tilde{u}_r}{\tilde{r}^2}\right] - \tilde{u}_r = 0, \tag{9}$$

$$-\frac{\partial \tilde{p}}{\partial \tilde{z}} + \frac{1}{Ha^2}\left[\frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{r}}\left(\tilde{r}\frac{\partial \tilde{u}_z}{\partial \tilde{r}}\right) + \frac{\partial^2 \tilde{u}_z}{\partial \tilde{z}^2}\right] = 0, \tag{10}$$

$$\begin{array}{lll} \tilde{u}_r = 0, \quad \tilde{u}_z = 1 & \text{at} & \tilde{r}^2 + \tilde{z}^2 = 1, \\ \tilde{u}_r \to 0, \quad \tilde{u}_z \to 0 & \text{for} & \tilde{r}^2 + \tilde{z}^2 \to \infty. \end{array} \tag{11}$$

We propose an asymptotic solution for $Ha \gg 1$.

If we assume $\tilde{u}_r = O(1)$, $\tilde{u}_z = O(1)$ and neglect $O(Ha^{-2})$ small terms in the momentum equation (9), (10), we have no solution satisfying boundary conditions. This fact suggest that the characteristic length-scale in $\tilde{z}$ direction is much larger than the sphere radius 1. Thus, we compress the vertical coordinate following the analyses by Chang [5] and Kyrlidis *et al.* [6]:

$$\tilde{r} = r, \quad \tilde{z} = Ha\, z, \quad \tilde{u}_r = Ha^{-1}u_r, \quad \tilde{u}_z = u_z, \quad \tilde{p} = Ha^{-1}p. \tag{12}$$

Neglecting $O(Ha^{-2})$ small terms after replacing variables in Eqs.(8)-(10), we obtain the following simplified equations.

$$\frac{1}{r}\frac{\partial r u_r}{\partial r} + \frac{\partial u_z}{\partial z} = 0, \tag{13}$$

$$-\frac{\partial p}{\partial r} - u_r = 0, \qquad -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) = 0. \tag{14}$$

Lorentz force balances with the horizontal component of the pressure gradient, while the viscous force due to share flow balances with the vertical component of the pressure gradient.

We introduce Stokes stream function ψ in order to reduce the number of equations:

$$u_r = -\frac{1}{r}\frac{\partial \psi}{\partial z}, \qquad u_z = \frac{1}{r}\frac{\partial \psi}{\partial r}. \tag{15}$$

Equation (13) is automatically satisfied. Taking rotation of Eqs. (14) and substituting Eq.(15) into it, we obtain the following equation without p.

$$\left[\left(\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r}\right)^2 - \frac{\partial^2}{\partial z^2}\right]\psi = 0. \tag{16}$$

Boundary conditions are

$$\psi = r^2/2 \qquad \text{at} \quad z = 0, \quad 0 \le r \le 1, \tag{17}$$

$$\begin{array}{ll} \psi = 0, \quad \partial\psi/\partial r = 0 & \text{at} \quad r = 0, \\ \partial\psi/\partial z \to 0 & \text{for} \quad z \to \pm\infty. \end{array} \tag{18}$$

In this coordinate system, the sphere surface $\tilde{r}^2 + \tilde{z}^2 = 1$ is reduced to the disk surface $z = \pm 0$, $0 \le r \le 1$.

3. Discussion about property of the solution We introduce the following variables in order to examine property of the function ψ.

$$f = \left(\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r}\right)\psi, \qquad g = \frac{\partial\psi}{\partial z}. \tag{19}$$

Equation (16) is reduced to the following equations.

$$\left[\left(\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r}\right) - \frac{\partial}{\partial z}\right](f+g) = 0, \quad \left[\left(\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r}\right) + \frac{\partial}{\partial z}\right](f-g) = 0. \tag{20}$$

The parabolic differential equations (20) show that the function $(f+g)$ spatially develops upward, while the function $(f-g)$ spatially develops downward. Considering the problem in infinite space, there is no disturbance coming from infinity. All the disturbances are submitted from the sphere and spatially develop upward and downward according to Eqs.(20).

From the above discussion, we decide an additional boundary condition:

$$u_z = 0 \qquad \text{at} \quad z = 0, \quad r > 1, \tag{21}$$

because no disturbance is submitted sideward in the horizontal plane including the sphere. The discontinuity of u_z at $(r, z) = (1, 0)$ is caused by the approximation neglecting $O(Ha^{-2})$ small terms. This suggests existence of a thin boundary layer around this point [5],[6].

The above thin boundary layer has negligibly small contribution to flow rate. Consequently, Eq.(21) is reduced to

$$\psi = 1/2 \qquad \text{at} \quad z = 0, \quad r > 1. \tag{22}$$

Furthermore, taking account of $u_r = 0$ at infinity, the following condition is required for arbitrary z.

$$\psi \to 1/2 \qquad \text{for} \quad r \to \infty. \tag{23}$$

The boundary conditions for ψ are given by Eqs.(17), (18), (22) and (23).

4. MHD Oseenlet The following solution satisfies Eq.(16).

$$\psi_{\mathrm{M}} = \frac{1}{2}\left[1 - \exp\left(-\frac{r^2}{4|z|}\right)\right]. \tag{24}$$

This solution satisfies boundary conditions (18), (22) and (23), though it does not satisfy the other one Eq.(17). This solution gives asymptotic behavior of far wake at $|z| \gg 1$ [5], as is shown in section 5.

We derive velocity, pressure and current density from Eq.(24) as follows.

$$\begin{aligned} u_{r\mathrm{M}} &= \frac{r}{8z|z|}\exp\left(-\frac{r^2}{4|z|}\right), \qquad u_{z\mathrm{M}} = \frac{1}{4|z|}\exp\left(-\frac{r^2}{4|z|}\right), \\ p_{\mathrm{M}} &= \frac{1}{4z}\exp\left(-\frac{r^2}{4|z|}\right), \qquad j_{\theta\mathrm{M}} = -\frac{r}{8z|z|}\exp\left(-\frac{r^2}{4|z|}\right), \end{aligned} \tag{25}$$

where dimensionless current density j_θ is defined as $\hat{j}_\theta = (\sigma\eta)^{1/2}a^{-1}U_0 j_\theta$. Figure 3 – 6 shows streamlines, distributions of $u_{z\mathrm{M}}$, $u_{r\mathrm{M}}$ and p_{M}, respectively. Left-hand side of these figures are mirror extensions for intuition. Distribution of $j_{\theta\mathrm{M}}$ is the same as that of $u_{r\mathrm{M}}$ except for the sign. The velocity $u_{z\mathrm{M}}$ above or below the rising body decrease with $\frac{1}{4}|z|^{-1}$ in far wake $|z| \gg 1$. The location at the maximum $|j_{\theta\mathrm{M}}|$ in each horizontal plane is $r = \sqrt{2|z|}$.

The solution (24) and (25) is one of the singularities such as stokeslet, oseenlet and source doublet [7]. This function is practically equal to the superposition of two oseenlets:

$$\begin{aligned} \psi_{\mathrm{o}+} + \psi_{\mathrm{o}-} &= \left(1 + \frac{\tilde{z}}{\sqrt{\tilde{r}^2 + \tilde{z}^2}}\right)\left[1 - \exp\left(-k\sqrt{\tilde{r}^2 + \tilde{z}^2} + k\tilde{z}\right)\right] \\ &\quad + \left(1 - \frac{\tilde{z}}{\sqrt{\tilde{r}^2 + \tilde{z}^2}}\right)\left[1 - \exp\left(-k\sqrt{\tilde{r}^2 + \tilde{z}^2} - k\tilde{z}\right)\right] \\ &= 2 - 2\exp\left(-k\sqrt{\tilde{r}^2 + \tilde{z}^2}\right)\left[\cosh(k\tilde{z}) + \frac{\tilde{z}}{\sqrt{\tilde{r}^2 + \tilde{z}^2}}\sinh(k\tilde{z})\right]. \end{aligned}$$

In case of ordinary oseenlet, the parameter k is replaced with a half of Reynolds number. Here, we replace k with $Ha/2$. Using the compressed coordinate (12), we obtain the following approximation.

$$\begin{aligned} \frac{\psi_{\mathrm{o}+} + \psi_{\mathrm{o}-}}{2} &\simeq 1 - \exp\left[-\frac{Ha^2|z|}{2}\left(1 + \frac{r^2}{2Ha^2z^2}\right)\right] \\ &\quad \times \left[\cosh\left(\frac{Ha^2 z}{2}\right) + \frac{z}{|z|}\left(1 - \frac{r^2}{2Ha^2z^2}\right)\sinh\left(\frac{Ha^2 z}{2}\right)\right] \\ &= 1 - \exp\left(-\frac{r^2}{4|z|}\right)\left[1 - \frac{r^2}{2Ha^2z|z|}\exp\left(-\frac{Ha^2|z|}{2}\right)\sinh\left(\frac{Ha^2 z}{2}\right)\right] \\ &\simeq 1 - \exp\left(-\frac{r^2}{4|z|}\right), \end{aligned} \tag{26}$$

where Ha is assumed to be much larger then 1. According to the above relation, we call the solution (24) and (25) "MHD oseenlet."

MHD oseenlet gives double wake as shown in Fig. 3. Double wake along the magnetic field is unique to MHD flows [8]. It is noticeable that the flow rate of MHD oseenlet across any horizontal plane is a finite value π ($\pi a^2 U_0$ in dimensional

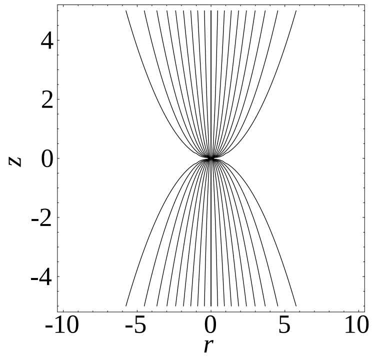

Figure 3: Streamlines of MHD oseenlet.

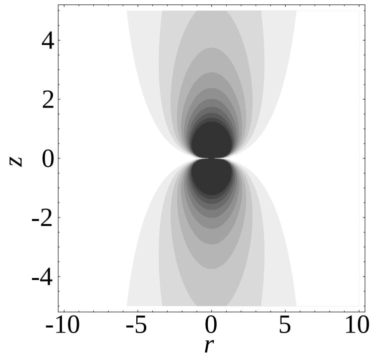

Figure 4: Distribution of u_{z_M} of MHD oseenlet. (White: smaller than 0.01, Black: greater than 0.2.)

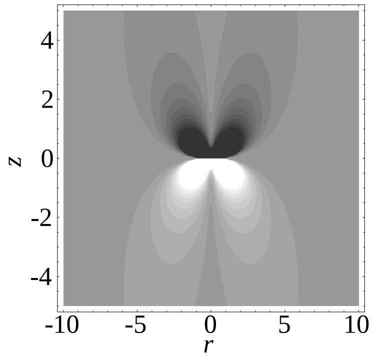

Figure 5: Distribution of u_{r_M} of MHD oseenlet. (White: smaller than -0.1, Black: greater than 0.1.)

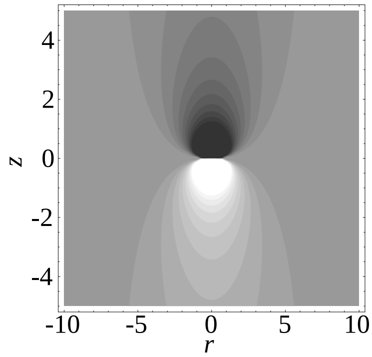

Figure 6: Distribution of p_M of MHD oseenlet. (White: smaller than -0.2, Black: greater than 0.2.)

form). This fact contrasts with divergent flow rate for stokeslet and zero flow rate for source doublet.

In the same manner as the other singularities, the derivatives of MHD oseenlet

$$\psi_\mathrm{M}^{(n)} = \frac{\partial^n}{\partial z^n} \exp\left(-\frac{r^2}{4|z|}\right) \qquad (n = 1, 2, 3, \cdots) \tag{27}$$

also satisfy Eq.(16). Furthermore, non-axisymmetric solution

$$u_{z_\mathrm{M}}^{(k,m,n)} = \frac{\partial^{k+m+n}}{\partial x^k \partial y^m \partial z^n} \frac{1}{|z|} \exp\left(-\frac{x^2+y^2}{4|z|}\right) \qquad \begin{pmatrix} k = 1, 2, 3, \cdots \\ m = 1, 2, 3, \cdots \\ n = 1, 2, 3, \cdots \end{pmatrix} \tag{28}$$

satisfy the following equation:

$$\left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 - \frac{\partial^2}{\partial z^2}\right] u_z = 0, \tag{29}$$

which is a replacement of Eq.(16). Linear combination of MHD oseenlet and its derivatives constructs solutions for various boundary conditions.

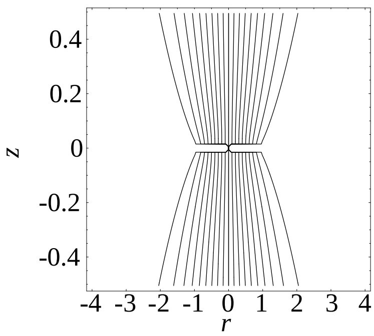

Figure 7: Streamlines of near wake.

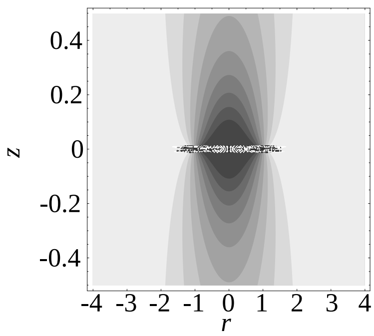

Figure 8: Distribution of u_z in near wake (white: $u_z < 0$, Black: $u_z > 1$).

5. Near wake of the rising sphere We assume the solution around a sphere as follows.

$$\psi = \frac{1}{2} - \sum_{n=0}^{\infty} \frac{c_n^+}{2} \frac{\partial^n}{\partial z^n} \exp\left(-\frac{r^2}{4z}\right), \quad u_z = \sum_{n=0}^{\infty} c_n^+ \frac{\partial^n}{\partial z^n} \frac{1}{4z} \exp\left(-\frac{r^2}{4z}\right) \qquad \text{for } z > 0,$$

$$\psi = \frac{1}{2} - \sum_{n=0}^{\infty} \frac{c_n^-}{2} \frac{\partial^n}{\partial z^n} \exp\left(\frac{r^2}{4z}\right), \qquad u_z = \sum_{n=0}^{\infty} c_n^- \frac{\partial^n}{\partial z^n} \frac{(-1)}{4z} \exp\left(\frac{r^2}{4z}\right) \qquad \text{for } z < 0. \tag{30}$$

This solution satisfies the governing equation (16) or (29). In addition, the boundary condition (17) or

$$\lim_{|z|\to 0} u_z = 1 \qquad \text{for } 0 \le r \le 1 \tag{31}$$

is required. Each term of the series in Eq.(30) has a singular point at $(r, z) = (0, 0)$. Consequently, we can not take termwise limit for $|z| \to 0$. Therefore, we require the following alternative condition on determination of the constant c_n^+, c_n^-.

$$\lim_{|z|\to 0} \int_0^1 u_z r^{2k+1}\,\mathrm{d}r = \int_0^1 r^{2k+1}\,\mathrm{d}r \qquad (k = 0, 1, 2, \cdots). \tag{32}$$

Substituting Eq.(30) into (32), We obtain c_n^+ and c_n^- as follows.

$$c_n^+ = \frac{1}{2^{2n} n!(n+1)!}, \qquad c_n^- = \frac{(-1)^n}{2^{2n} n!(n+1)!}. \tag{33}$$

Figure 7 and 8 show streamlines and distribution of u_z around the sphere. (The sphere is reduced to a disk at $z = 0$, $0 \le r \le 1$ in the compressed coordinate r, z.) On drawing these figures, we truncate the series after 51 terms. Therefore, these figures include remarkable truncation error in the vicinity of $z = 0$. There is a cylindrical share layer at $r = 1$, $|z| \ll 1$ as mentioned in Ref.[3]–[6]. Velocity u_z in $-0.1(1 - r^2) < z < 0.1(1 - r^2)$ is greater than 0.9. Here, we call this region the conical-shape near wake. Fluid in the near wake moves at almost the same velocity with the rising sphere. In dimensional space $\hat{r}\hat{\theta}\hat{z}$, vertical height $\pm 0.1 Ha\, a$ of the conical-shape near wake is much larger than its radius a.

The nth derivative of MHD oseenlet has the factor $|z|^{-n}$. Consequently, higher order derivatives rapidly decrease with $|z|$. Therefore, the flow field is reduced to MHD oseenlet (the leading term of the series (30)) in far wake $|z| \gg 1$. This result agrees with the result of Chang [5].

6. Drag of the sphere We try to derive the drag of the sphere from the above solution. Let us integrate the left-hand side of Eq.(14) over a volume $0 \le r \le r_1$, $-z_1 \le z \le z_1$ which includes the sphere. Using Gauss's divergence theorem, we obtain

$$-\int_0^{r_1} 2\pi(p)_{z=z_1} r\,\mathrm{d}r + \int_0^{r_1} 2\pi(p)_{z=-z_1} r\,\mathrm{d}r + \int_{-z_1}^{z_1} 2\pi \left(r\frac{\partial u_z}{\partial r}\right)_{r=r_1} \mathrm{d}z. \qquad (34)$$

This shows z component of inward momentum flux to the volume. In case of steady flow, the sphere sucks the same amount of momentum flux from the fluid around it. (The coordinate system moving with the sphere is suitable for this discussion. But, we omit coordinate transformation because this problem has no essential difference between the moving system and the rest system.) Because drag D is outward momentum flux form the sphere to the fluid, $-D$ is equal to Eq.(34). Velocity u_z is an even function of z, while pressure p is an odd function of z. Therefore,

$$D = 4\pi \int_0^{r_1} (p)_{z=z_1} r\,\mathrm{d}r - 4\pi \int_0^{z_1} \left(r\frac{\partial u_z}{\partial r}\right)_{r=r_1} \mathrm{d}z. \qquad (35)$$

Taking account of $p = u_z$ for $z > 0$, we obtain

$$\begin{aligned} D &= -2\pi \left[\exp\left(-\frac{r^2}{4z_1}\right)\right]_0^{r_1} + 2\pi \left[\exp\left(-\frac{{r_1}^2}{4z}\right)\right]_0^{z_1} \\ &= 2\pi \end{aligned} \qquad (36)$$

from the near wake solution (30), where derivatives of MHD oseenlet have no contribution. Drag $\hat{D}$ in dimensional form is $2\pi\eta a U_0 Ha$. This result is identical to the result of Chester [3].

7. Conclusions Flow field of electrically conducting fluid around a rising sphere is discussed when uniform magnetic field is aligned with the gravity. We propose an asymptotic solution for $Ha \gg 1$.

- We derive MHD oseenlet from a simplified momentum equation with Lorentz force term. It is practically equal to the superposition of two ordinary oseenlets.
- MHD oseenlet gives double wake unique to MHD flows. Flow rate of MHD oseenlet across any horizontal plane is a finite value $\pi a^2 U_0$. This fact contrasts with divergent flow rate for stokeslet and zero flow rate for source doublet.
- We obtain the flow field around a sphere in form of a series of MHD oseenlet and its derivatives.
- The fluid in the conical-shape near wake moves with the rising sphere. The vertical height $\pm 0.1 Ha\, a$ of the near wake is much larger than its radius a.
- Flow field is reduced to MHD oseenlet in far wake.

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